Take Home Assignment 1

1. A cylindrical container is to be manufactured with a volume of 200 cubic centimeters. The cylinder will be cut from sheets of stainless steel that cost \$50.00/ m^2, and the caps will be cut from sheets of a different grade of stainless steel that cost \$75.00/ m^2. Find the dimensions of the can that minimize the cost of the materials.

Find the rate of change dC/dV of the (minimal) materials-cost (C) of the container with respect to its volume (V).

2. Find the average distance to the origin of points in the ball

$$x^2 + y^2 + z^2 \leq R^2.$$

3. Find the *singular value decomposition* of the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}.$$

4. Find an orthogonal transformation of $\mathbb{R}^3$ that transforms the quadratic form

$$Q(x, y, z) = x^2 + 2xy + 4xz + 2y^2 + 2yz + z^2$$

to the diagonal form

$$\mathcal{Q}(u, v, w) = \alpha u^2 + \beta v^2 + \gamma w^2$$

(and find the coefficients α, β and γ).

5. Find the unit tangent, normal and binormal, $\hat{\mathbf{t}}, \hat{\mathbf{n}}, \hat{\mathbf{b}}$, and the curvature κ as functions of t for the helix

$$\mathbf{r}(t) = a\cos(\omega t)\mathbf{i} + a\sin(\omega t)\mathbf{j} + bt\mathbf{k}.$$

6. A function $\varphi(x, y, z)$ (a scalar field) is called *radial* if it is constant on spheres around the origin, i.e., $\varphi(x, y, z) = \varphi(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$.

a. What is the Laplacian of a radial function? (Suggestion: use spherical coordinates).

b. A function $u(x, y, z)$ is *harmonic* if $\nabla^2 u = 0$. Show that a radial harmonic function $u(x, y, z)$ defined in all of $\mathbb{R}^3$ must be constant.