Take Home Assignment 1

1. A cylindrical container is to be manufactured with a volume of 200 cubic centimeters. The cylinder will be cut from sheets of stainless steel that cost $50.00/ m², and the caps will be cut from sheets of a different grade of stainless steel that cost $75.00/ m². Find the dimensions of the can that minimize the cost of the materials.

Find the rate of change $dC/dV$ of the (minimal) materials-cost $(C)$ of the container with respect to its volume $(V)$.

2. Find the average distance to the origin of points in the ball

$$x^2 + y^2 + z^2 \leq R^2.$$ 

3. Find the singular value decomposition of the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}.$$ 

4. Find an orthogonal transformation of $\mathbb{R}^3$ that transforms the quadratic form

$$Q(x, y, z) = x^2 + 2xy + 4xz + 2y^2 + 2yz + z^2$$

to the diagonal form

$$Q(u, v, w) = \alpha u^2 + \beta v^2 + \gamma w^2$$

(and find the coefficients $\alpha$, $\beta$ and $\gamma$).

5. Find the unit tangent, normal and binormal, $\hat{t}$, $\hat{n}$, $\hat{b}$, and the curvature $\kappa$ as functions of $t$ for the helix

$$r(t) = a \cos(\omega t)i + a \sin(\omega t)j + bt k.$$ 

6. A function $\varphi(x, y, z)$ (a scalar field) is called radial if it is constant on spheres around the origin, i.e., $\varphi(x, y, z) = \varphi(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$.

a. What is the Laplacian of a radial function? (Suggestion: use spherical coordinates).

b. A function $u(x, y, z)$ is harmonic if $\nabla^2 u = 0$. Show that a radial harmonic function $u(x, y, z)$ defined in all of $\mathbb{R}^3$ must be constant.